Appendix A
Arithmetic Review

The following is intended as a quick refresher of some of the simple arithmetic operations you learned in high school but probably have not used since. Although some of what follows will seem so obvious that you wonder why it is included, people sometimes forget the most obvious things. A more complete review can be found at http://www.uvm.edu/~dhowell/fundamentals9/ArithmeticReview/review_of_arithmetic_revised.html and I recommend that anyone who is not sure of the arithmetic skills should look at that.

One of the things that students never seem to learn is that it is easy to figure out most of these principles for yourself. For example, if you can't remember whether

$$\frac{18.1}{28.6 + 32.7} \text{ can be reduced to } \frac{18.1}{28.6} + \frac{18.1}{32.7}$$

(it cannot, but it is one of the foolish things that I can never keep in my head), try it out with very simple numbers. Thus,

$$\frac{2}{1 + 4} = \frac{2}{5} = .4$$

is obviously not the same as

$$\frac{2}{1} + \frac{2}{4} = 2.5$$

It is often quicker to check on a procedure by using small numbers than by looking it up.

Standard Symbols and Basic Information

Numerator	The thing on the top
Denominator	The thing on the bottom
a/b	a = Numerator; b = Denominator
$+, -, \times, \div$ (or /)	Symbols for addition, subtraction, multiplication, and division; called *operators*
$X = Y$	X equals Y
$X \approx Y$ or $X \simeq Y$	X approximately equal to Y
$X \neq Y$	X unequal to Y
$X < Y$	X less than Y (*Hint:* The smaller end points to the smaller number.)
$X \leq Y$	X less than or equal to Y
$X > Y$	X greater than Y
$X \geq Y$	X greater than or equal to Y
$X < Y < Z$	X less than Y less than Z (i.e., Y is between X and Z)
$X \pm Y$	X plus or minus Y
$\lvert X \rvert$	Absolute value of X—ignore the sign of X
$\frac{1}{X}$	The reciprocal of X
X^2	X squared
X^n	X raised to the nth power
$\sqrt{X} = X^{1/2}$	Square root of X

Addition and Subtraction

$8 - 12 = -4$	To subtract a larger number from a smaller one, subtract the smaller from the larger and make the result negative.
$-8 + 12$ $= 12 - 8 = 4$	The order of operations is not important.

Multiplication and Division

$2(3)(6)$ $= 2 \times 3 \times 6$	If no operator appears before a set of parentheses, multiplication is implied.
$2 \times 3 \times 6$ $= 2 \times 6 \times 3$	Numbers can be multiplied in any order.
$\frac{2 \times 8}{4}$ $= \frac{2}{4} \times 8$ $= 2 \times \frac{8}{4}$ $= \frac{16}{4} = 4$	Division can take place in any order.
$7 \times 3 + 6$ $= 21 + 6 = 27$	Multiply or divide *before* you add or subtract the result. But for the same operators, work from left to right [e.g., $8 \div 2 \div 4 = (8 \div 2) \div 4$, not $8 \div (2 \div 4)$].
$2 \times 3 = 6$ $(-2)(-3) = 6$ $\frac{6}{3} = 2$ $\frac{-6}{-3} = 2$	Multiplication or division of numbers with the *same* sign produces a positive answer.
$(-2)3 = -6$ $\frac{-6}{3} = -2$	Multiplication or division of numbers with *opposite* signs produces a negative answer.
$(-2)(3)(-6)(-4)$ $= (-6)(24)$ $= -144$	With several numbers having different signs, work in pairs to get the correct sign.

Parentheses

$2(7 - 6 + 3) =$ $2(4) = 8$	When multiplying, either perform the operations inside parentheses before multiplying, or multiply *each* element within the parentheses and then sum.

or

$2(7) + 2(-6) + 2(3)$
$= 14 - 12 + 6 = 8$

$2(7 - 6 + 3)^2$
$= 2(4)^2 = 2(16)$
$= 32$

When the parenthetical term is raised to a power, perform the operations inside the parentheses, raise the result to the appropriate power, and then carry out the other operations.

Fractions

$\frac{1}{5} = .20$

To convert to a decimal, divide the numerator by the denominator.

$\frac{4}{3}$

The reciprocal of $\frac{3}{4}$. To take the reciprocal of a fraction, stand it on its head.

$3 \times \frac{6}{5} = \frac{3 \times 6}{5}$
$= \frac{18}{5} = 3.6$

To multiply a fraction by a whole number, multiply the numerator by that number.

$\frac{3}{5} \times \frac{6}{7} \times \frac{1}{2}$
$= \frac{3 \times 6 \times 1}{5 \times 7 \times 2}$
$= \frac{18}{70} = .26$

To multiply a series of fractions, multiply numerators together and multiply denominators together.

$\frac{1}{3} + \frac{4}{3} = \frac{5}{3} = 1.67$

To add fractions with the *same* denominator, add the numerators and divide by the common denominator.

$\frac{1}{6} + \frac{4}{3} = \frac{1}{6} + \frac{8}{6} = \frac{9}{6}$
$= 1.5$

To add fractions with *different* denominators, multiply the numerator and the denominator by a constant to equate the denominators and follow the previous rule.

$\frac{8}{13} + \frac{12}{25}$

This is a more elaborate example of the same rule.

$= \left(\frac{25}{25} \times \frac{8}{13}\right) + \left(\frac{13}{13} \times \frac{12}{25}\right)$

$$= \frac{200}{325} + \frac{156}{325}$$

$$= \frac{356}{325} = 1.095$$

$$\frac{8}{1/3} = 8\left(\frac{3}{1}\right) = 24$$

To divide by a fraction, multiply by the reciprocal of that fraction.

Algebraic Operations

Most algebraic operations boil down to moving things from one side of the equation to the other. Mathematically, the rule is that whatever you do to one side of the equation you must do to the other side.

Solve the following equation for X:

$$3 + X = 8$$

We want X on one side and the answer on the other. All we have to do is to subtract 3 from both sides to get

$$3 + X - 3 = 8 - 3$$
$$X = 5$$

If the equation had been

$$X - 3 = 8$$

we would have added 3 to both sides:

$$X - 3 + 3 = 8 + 3$$
$$X = 11$$

For equations involving multiplication or division, we follow the same principle:

$$2X = 21$$

Dividing both sides by 2, we have

$$\frac{2X}{2} = \frac{21}{2}$$
$$X = 10.5$$

and

$$\frac{X}{7} = 13$$

$$\frac{7X}{7} = 7(13)$$

$$X = 91$$

Personally, I prefer to think of things in a different, but perfectly equivalent, way. When you want to get rid of something that has been added (or subtracted) to (or from) one side of the equation, move it to the other side and reverse the sign:

$$3 + X = 12 \qquad \text{or} \qquad X - 7 = 19$$

$$X = 12 - 3 \qquad\qquad X = 19 + 7$$

When the thing you want to get rid of is in the numerator, move it to the other side and put it in the denominator:

$$7.6X = 12$$

$$X = \frac{12}{7.6}$$

When the thing you want to get rid of is in the denominator, move it to the numerator on the other side and multiply:

$$\frac{X}{8.9} = 14.6$$

$$X = 14.6(8.9)$$

Notice that with more complex expressions, you must multiply (or divide) everything on the other side of the equation. Thus,

$$7.6X = 12 + 8$$

$$X = \frac{12 + 8}{7.6}$$

For complex equations, just work one step at a time:

$$7.6(X + 8) = \frac{14}{7} - 5$$

First, get rid of the 7.6:

$$X + 8 = \frac{14/7 - 5}{7.6}$$

Now get rid of the 8:

$$X = \frac{14/7 - 5}{7.6} - 8$$

Now clean up the messy fraction:

$$X = \frac{2 - 5}{7.6} - 8 = \frac{-3}{7.6} - 8 = -.395 - 8 = -8.395$$