APPENDIX A

Mathematics Review

This review briefly covers the mathematics needed to understand the statistics presented in this text.

Statistical Symbols

A **symbol**, sometimes called a **sign**, is a letter or character used to represent something. The following table presents the common symbols used in statistics.

Symbol and Example	Description
X or X_i	X is used to represent a score or a measurement obtained from a subject. The subscript i, when used, indicates the score of a particular subject. For example, X_1 represents the score of subject number 1.
$=$	Equals sign.
$X_1 = 3$	The score of subject 1 equals 3.
$X_5 = 27$	The score of subject 5 equals 27.
$a = 2$	The value of a equals 2.
$\neq$	Does not equal sign.
$6 \neq 3$	6 is not equal to 3.
$+$	Plus sign. This sign indicates that the numbers joined by the + should be added. The result of addition is called the *sum*. A statement such as "The sum of 5 and 1" indicates that the numbers are to be added together.
$5 + 1 = 6$	The sum of 5 plus 1 equals 6.
$5 + 1 + 6 = 12$	The sum of 5 plus 1 plus 6 equals 12.
$a + b$	a plus b: The value of b is to be added to the value of a.
$-$	Minus sign. This sign indicates that the number that follows the $-$ should be subtracted from the number preceding the $-$ sign. The result of subtraction is called the *difference*. A statement such as "The difference of 5 and 1" indicates that one number is to be subtracted from the other.
$5 - 1 = 4$	The difference of 5 minus 1 is 4.
$a - b$	a minus b: The value of b is to be subtracted from the value of a.
( )( ) or $\times$	Multiplication or times sign. The values in parentheses or separated by the $\times$ are multiplied. The result of multiplication is called a *product*.

$(2)(3) = 6$	The product of 2 multiplied by 3 is 6.
$2 \times 3 = 6$	The product of 2 multiplied by 3 is 6.
$(a)(b)$ or $a \times b$	a times b: The value of a is to be multiplied by the value of b. Exception to the rule: In a factorial design the interaction of factors A and B is indicated by $A \times B$. In this instance, the term should be read as "the A by B interaction." It does not mean that A is multiplied by B.
$\div$ or / or —	Division sign. The number preceding the $\div$ or the / or the number above the — (the *numerator*) is divided by the number following the $\div$ or the / or the number below the — (the *denominator*), respectively. The result of division is called the *quotient*.
$6 \div 2 = 3$	The quotient of 6 divided by 2 is 3.
$6/2 = 3$	The quotient of 6 divided by 2 is 3.
$\frac{6}{2} = 3$	The quotient of 6 divided by 2 is 3.
$a \div b$, or a/b, or $\frac{a}{b}$	The quotient of a divided by b.
$<$	Less than symbol. The number preceding the $<$ is less than the number following the $<$.
$2 < 3$	2 is less than 3.
$a < b$	The value of a is less than the value of b.
$X_3 < 10$	The score of subject 3 is less than 10.
$>$	Greater than symbol. The number preceding the $>$ is greater than the number following the $>$.
$3 > 2$	3 is greater than 2.
$a > b$	The value of a is greater than the value of b.
$X_3 > 10$	The score of subject 3 is greater than 10.
$\leq$	Less than or equal to symbol. The number preceding the $\leq$ is less than or equal to the number following the $\leq$.
$2 \leq 3$	2 is less than or equal to 3.
$4 \leq 4$	4 is less than or equal to 4.
$a \leq b$	The value of a is less than or equal to the value of b.
$X_3 \leq 10$	The score of subject 3 is less than or equal to 10.
$\geq$	Equal to or greater than symbol. The number preceding the $\geq$ is equal to or greater than the number following the $\geq$.
$3 \geq 2$	3 is equal to or greater than 2.
$4 \geq 4$	4 is equal to or greater than 4.
$a \geq b$	The value of a is equal to or greater than the value of b.
$X_3 \geq 10$	The score of subject 3 is equal to or greater than 10.
$5 < X_1 < 10$	The greater than and less than symbols may be placed in one term as illustrated. This term is read as "The score of subject 1 is greater than 5 and less than 10."

$5 \leq X_1 \leq 10$	The equal to or greater than and less than or equal to symbols may also be placed in one term as illustrated. This term is read as "The score of subject 1 is equal to or greater than 5 and equal to or less than 10."
$\lvert\,\rvert$	Absolute value symbol. This symbol indicates that we ignore the + or − sign attached to a number.
$\lvert -6 \rvert = 6$	The absolute value of negative 6 is 6.
$\lvert +6 \rvert = 6$	The absolute value of positive 6 is 6.
$(\,)^2$	The number enclosed in parentheses is squared, or multiplied by itself. Sometimes the parentheses are not used, and the square indicator, ^{2}, simply follows the number to be squared.
$(5)^2 = 25$	5 squared, which is 5×5, equals 25.
$5^2 = 25$	5 squared equals 25.
$\sqrt{}$	Square root symbol. The $\sqrt{}$ indicates finding the number that, when multiplied by itself (i.e., when squared), equals the number under the $\sqrt{}$ symbol.
$\sqrt{25} = 5$	The square root of 25 equals 5, for 5 multiplied by itself (i.e., 5×5) equals 25.
$\sqrt{36} = 6$	The square root of 36 equals 6, for 6 multiplied by itself (i.e., 6×6) equals 36.
Σ	Summation symbol. The numbers following the Σ should be added together.
$\Sigma\,(3 + 2) = 5$	The sum of 3 plus 2 is 5.
$\Sigma\,(3 + 2 + 4) = 9$	The sum of 3 plus 2 plus 4 is 9.
ΣX or ΣX_i	The sum of X. Add all the scores that are represented by X. For example, if the scores of three subjects are $X_1 = 3$, $X_2 = 6$, and $X_3 = 4$, then $\Sigma X = 3 + 6 + 4 = 13$. Sometimes limits are placed on the summation sign, such as $\sum_{i=1}^{3} X_i$. These limits indicate the scores of subjects 1 to 3 should be added. If limits are not used, then ΣX means all the scores designated by X should be added.

Mathematical Operations

Negative Numbers

A negative number may occur as a result of subtraction, such as $6 - 8 = -2$. The 8, which is larger than the 6 by 2, results in a difference of -2 when it is subtracted from the 6. Negative numbers occur often in statistics and then are used in basic mathematical operations. If a number is not preceded by a minus sign, it is assumed to be positive.

Adding Negative Numbers

$6 + (-4) = 2$ Adding a negative number to a positive number is equivalent to subtracting the negative number from the positive number. Thus, $6 + (-4) = 6 - 4 = 2$.

$(-6) + (-4) = -10$ In this instance, the -4 is subtracted from the -6, or $(-6) + (-4) = -6 - 4 = -10$

Subtracting Negative Numbers

$6 - (-4) = 10$ Subtracting a negative number is equivalent to adding the absolute value of the negative number. Thus, $6 - (-4) = 6 + |-4| = 6 + 4 = 10$.

Multiplying Negative Numbers

$(-6)(-4) = 24$ If both numbers to be multiplied are negative, then the product is positive.

$(6)(-4) = -24$ If only one of the two numbers is negative, then the product is negative.

$(-6)(4) = -24$

Dividing Negative Numbers

$-10 \div 5 = -2$ If either the numerator or the denominator is negative, then the quotient is negative.

$10 \div -5 = -2$

$-10 \div -5 = 2$ If both the numerator and the denominator are negative, then the quotient is positive.

Fractions

The easiest approach to working with fractions is to convert the fraction to a decimal by dividing the numerator by the denominator, for example, $\frac{1}{4} = 0.25$, $\frac{2}{4} = 0.50$, and $\frac{3}{4} = 0.75$. If the decimal is to be involved in further calculations, then it should be carried to at least three decimal places (e.g., $\frac{4}{11} = 0.364$). Thus, $\frac{2}{5} + \frac{1}{8} + \frac{1}{3}$ equals $0.400 + 0.125 + 0.333 = 0.858$.

Proportions and Percents

Proportion

A proportion is a part of a whole. For example, if 100 people answer a questionnaire and 45 of them are males, then the proportion of male respondents is $\frac{45}{100}$ or .45. The proportion of female respondents is $\frac{55}{100}$ or .55.

Percent

A percent is formed when a proportion is multiplied by 100. Thus, a proportion of .45 equals $.45 \times 100$ or 45 percent. A proportion of .55 equals $.55 \times 100 = 55$ percent.

Order of Mathematical Operations

The following examples illustrate the order in which mathematical operations are performed with common statistical terms.

Term	Order of Mathematical Operations
$\sum(X - \overline{X})$	**1.** The mean is subtracted from each score in a set of scores.
	2. The differences are summed.
$\sum(X - \overline{X})^2$	**1.** The mean is subtracted from each score in a set of scores.
	2. Each difference is squared.
	3. The squared differences are summed.
$\sum X^2$	**1.** Each X value is squared.
	2. The squared X values are summed.
$(\sum X)^2$	**1.** The X values are summed.
	2. The sum of the X values (i.e., $\sum X$) is squared.
$\sum XY$	**1.** The corresponding X and Y values are multiplied for the set of scores.
	2. The multiplied XY values are summed for the set of scores.
$(\sum X)(\sum Y)$	**1.** The X values are summed to obtain $\sum X$.
	2. The Y values are summed to obtain $\sum Y$.
	3. The sum of the X values (i.e., $\sum X$) is multiplied by the sum of the Y variables (i.e., $\sum Y$).

Rounding Rules

There are several rules that are commonly used when numerical computations require rounding.

Rounding: Answers to Computations

Suppose that you obtained three scores on a task of recalling a list of words: 4, 4, and 3. To describe the typical response, you found the sum of these numbers, 11, and divided it by 3, or $\frac{11}{3} = 3.666\overline{6}$. The bar (–) over the last 6 indicates that the 6s continue endlessly. How many decimal places should you present in your answer?

One convention is that the final value of a computation should be rounded to two decimal places beyond the value to which the original scores were measured. In the example, the scores were given to the ones (or units) place; a person could obtain 3 correct answers or 4 correct answers, but not 3.2 or 4.6. Following this rule, the final value of the computation is rounded to two decimal places beyond the ones place, or to 3.67.

Others follow a convention of rounding the final value of a computation to one decimal place beyond the value to which the original scores were measured. Following this approach, the final value of the computation for the example is rounded to one decimal place beyond the ones place, or to 3.7. We follow this approach in text examples.

Rounding: Intermediate Steps in Computations

If computations require a number of steps prior to obtaining the final answer, then intermediate numerical values should be carried to at least two decimal places beyond the number of decimal places needed for the final answer. For example, if the value of $\frac{11}{3}$ were to enter into further computations, we would not round its value to 3.67. Rather, we carry it to 3.667 for the computations. This approach minimizes rounding error in computations.

Rounding Rules

Suppose that we have an answer to a computation such as $3.6AB$, where the A and B are possible numerical values. For example, if $A = 2$ and $B = 9$, then $3.6AB = 3.629$. We wish to round this number to two decimal places.

Several rules typically are followed when rounding numbers:

- If the number represented by the letter B is greater than 5, increase A by 1 and drop the B value. Following this rule, 3.629 (here $A = 2$ and $B = 9$) is rounded to 3.63.
- If the number represented by the letter B is less than 5, leave the A value as is and drop the B value. Following this rule, 3.623 (here $A = 2$ and $B = 3$) is rounded to 3.62.
- If the number represented by the letter B is exactly 5, increase A by 1 if A is an odd number, but leave its value as is if it is an even number; then drop B. Following this rule, 3.635 (here $A = 3$ and $B = 5$) is rounded to 3.64, but 3.625 (here $A = 2$ and $B = 5$) is rounded to 3.62. Notice, however, that if the number is 3.6251 it is rounded to 3.63, not 3.62.