APPENDIX D

Commonly Used Formulas

Frequency Distributions

Ungrouped Frequency Distributions

rf of a score $=$ f of a score$/N$

$\%f$ of a score $=$ (rf of a score) $\times$ 100

Grouped Frequency Distributions

$$\text{size of class interval} = i = \frac{X_{\text{highest}} - X_{\text{lowest}}}{\text{number of intervals}}$$

$$rf \text{ of scores in an interval} = \frac{f \text{ of scores in interval}}{N}$$

$\%f$ of scores in an interval $=$ (rf of scores in interval) $\times$ 100

Percentile Rank

$$P_X = \frac{cf_L + [(X - X_L)/i]\,f_i}{N} \times 100$$

Percentile of a Score

$$X_p = X_L + \left(\frac{P(N) - cf_L}{f_i}\right)i$$

Measures of Central Tendency

Median

$$Mdn = X_{.50} = X_L + \left(\frac{.50\,(N) - cf_L}{f_i}\right)i$$

Population Mean

$$\mu = \frac{\sum X}{N_{\text{population}}}$$

Sample Mean

$$\overline{X} = \frac{\sum X}{N}$$

Measures of Variability

Range

Range $= X_{\text{highest URL}} - X_{\text{lowest LRL}}$

Variance

Population Variance

$$\sigma^2 = \frac{\Sigma(X - \mu)^2}{N_{\text{population}}}$$

Variance of a Sample

$$S^2 = \frac{\Sigma(X - \overline{X})^2}{N}$$

Estimated Population Variance

Definitional formula: $s^2 = \dfrac{\Sigma(X - \overline{X})^2}{(N - 1)}$

Sum of squares formula: $s^2 = \dfrac{SS}{(N - 1)}$

Standard Deviation

Population Standard Deviation

$$\sigma = \sqrt{\sigma^2}$$

Standard Deviation of a Sample

$$S = \sqrt{S^2} = \sqrt{\frac{\Sigma(X - \overline{X})^2}{N}}$$

Estimated Population Standard Deviation

Definitional formula: $s = \sqrt{\dfrac{\Sigma(X - \overline{X})^2}{(N - 1)}}$

Sum of squares formula: $s = \sqrt{\dfrac{SS}{(N - 1)}}$

Standard Error of the Mean

σ Known

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}}$$

Estimated from *s*

$$s_{\overline{X}} = \frac{s}{\sqrt{N}}$$

Standard Error of the Difference between Means

σ Known

$$\sigma_{\overline{X}_1-\overline{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Estimated from s_1^2 and s_2^2

$$s_{\overline{X}_1-\overline{X}_2} = \sqrt{\left[\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}\right]\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}$$

Probability

Probability (p) of Occurrence of an Event

$$p(\text{event}) = \frac{\text{Number of outcomes composing the event}}{\text{Total number of possible outcomes}}$$

z Scores and Standard Scores

Single Score

$$z = \frac{X - \mu}{\sigma}$$

Standard Score

$$z = \frac{X - \overline{X}}{S} \quad \textbf{or} \quad z = \frac{X - \overline{X}}{\sqrt{\dfrac{\Sigma(X - \overline{X})^2}{N}}}$$

Sample Mean

$$z = \frac{(\overline{X} - \mu)}{\sigma_{\overline{X}}}$$

t Tests

One-Sample t Test

$$t = \frac{\overline{X} - \mu}{s_{\overline{X}}} \quad \textbf{or} \quad t = \frac{\overline{X} - \mu}{s/\sqrt{N}},\ df = N - 1$$

t Test for Independent Groups

$$\text{Definitional formula: } t_{\text{ind}} = \frac{\overline{X}_1 - \overline{X}_2}{s_{\overline{X}_1-\overline{X}_2}},\ df = N - 2$$

$$\text{Computational formula: } t_{\text{ind}} = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\left[\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}\right]\left[\dfrac{1}{n_1} + \dfrac{1}{n_2}\right]}},\ df = N - 2$$

t Test for Related Scores

$$t_{\text{rel}} = \frac{\overline{X}_1 - \overline{X}_2}{s_{\overline{D}}},\ df = N_{\text{pairs}} - 1, \text{ or}$$

$$t_{\text{rel}} = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{s_{\overline{D}}^2}{N_{\text{pairs}}}}},\ df = N_{\text{pairs}} - 1$$

Analysis of Variance

One-Factor Between-Subjects Design

Source	*SS*	*df*[a]	*MS*	*F*
Factor *A*	$\Sigma\Sigma(\overline{X}_A - \overline{X}_G)^2$	$a - 1$	SS_A/df_A	MS_A/MS_{Error}
Error	$\Sigma\Sigma(X - \overline{X}_A)^2$	$N - a$	$SS_{\text{Error}}/df_{\text{Error}}$	
Total	$\Sigma\Sigma(X - \overline{X}_G)^2$	$N - 1$	Not calculated	

[a]*a* = number of levels of factor *A*, *N* = total number of scores.

Two-Factor Between-Subjects Design

Source	*SS*	*df*[a]	*MS*	*F*
Factor *A*	$\Sigma\Sigma(\overline{X}_A - \overline{X}_G)^2$	$a - 1$	$\frac{SS_A}{df_A}$	$\frac{MS_A}{MS_{\text{Error}}}$
Factor *B*	$\Sigma\Sigma(\overline{X}_B - \overline{X}_G)^2$	$b - 1$	$\frac{SS_B}{df_B}$	$\frac{MS_B}{MS_{\text{Error}}}$
Interaction of *A* and *B*	$\Sigma\Sigma(\overline{X}_{AB} - \overline{X}_A - \overline{X}_B + \overline{X}_G)^2$	$(a - 1)(b - 1)$	$\frac{SS_{A\times B}}{df_{A\times B}}$	$\frac{MS_{A\times B}}{MS_{\text{Error}}}$
Error	$\Sigma\Sigma(X - \overline{X}_{AB})^2$	$ab(n_{AB} - 1)$	$\frac{SS_{\text{Error}}}{df_{\text{Error}}}$	
Total	$\Sigma\Sigma(X - \overline{X}_G)^2$	$N - 1$	Not calculated	

[a]*a* = number of levels of factor *a*; *b* = number of levels of factor *B*; n_{AB} = number of scores in each cell; *N* = total number of scores.

One-Factor Within-Subjects Design

Source	*SS*	*df*[*]	*MS*	*F*
Factor *A*	$\Sigma\Sigma(\overline{X}_A - \overline{X}_G)^2$	$a - 1$	$\frac{SS_A}{df_A}$	$\frac{MS_A}{MS_{A\times S}}$
Factor *S*	$\Sigma\Sigma(\overline{X}_S - \overline{X}_G)^2$	$n_A - 1$	$\frac{SS_S}{df_S}$	
$A \times S$	$\Sigma\Sigma(X - \overline{X}_A - \overline{X}_S + \overline{X}_G)^2$	$(a - 1)(n_A - 1)$	$\frac{SS_{A\times S}}{df_{A\times S}}$	
Total	$\Sigma\Sigma(X - \overline{X}_G)^2$	$N - 1$	Not calculated	

[*]*a* = number of levels of factor *A*; n_A = number of scores in a treatment condition or, equivalently, the number of subjects; *N* = total number of scores.

Tukey HSD Multiple Comparison Tests

For One-Factor Between-Subjects Designs

$$CD = q\sqrt{\frac{MS_{\text{Error}}}{n_A}}$$

For One-Factor Within-Subjects Designs

$$CD = q\sqrt{\frac{MS_{A \times S}}{n_A}}$$

For Simple Effects in a Factorial Between-Subjects Design

$$CD = q\sqrt{\frac{MS_{\text{Error}}}{n_{AB}}}$$

Effect Size Measures

Eta Squared for *t* Test

$$\eta^2 = \frac{t_{\text{obs}}^2}{t_{\text{obs}}^2 + df}$$

Eta Squared for One-Factor Between-Subjects Analysis of Variance

$$\eta^2 = \frac{SS_A}{SS_{\text{Total}}}$$

or

$$\eta^2 = \frac{(df_A)(F_{\text{obs}})}{(df_A)(F_{\text{obs}}) + df_{\text{Error}}}$$

Eta Squared for One-Factor Within-Subjects Analysis of Variance

$$\eta^2 = \frac{SS_A}{SS_A + SS_{A \times S}}$$

Eta Squared for Two-Factor Between-Subjects Analysis of Variance

$$\eta^2 = \frac{SS_A}{SS_{\text{Total}}} \quad \text{for factor } A$$

$$\eta^2 = \frac{SS_B}{SS_{\text{Total}}} \quad \text{for factor } B$$

$$\eta^2 = \frac{SS_{A \times B}}{SS_{\text{Total}}} \quad \text{for the interaction of factors } A \text{ and } B$$

Correlation

Pearson Correlation Coefficient

Definitional formula: $r = \dfrac{\Sigma(X - \overline{X})(Y - \overline{Y})}{\sqrt{[\Sigma(X - \overline{X})^2][\Sigma(Y - \overline{Y})^2]}}$

Cross products formula: $r = \dfrac{CP_{XY}}{\sqrt{(SS_X)(SS_Y)}}$

Standard score formula: $r = \dfrac{\Sigma(z_X z_Y)}{N_{\text{pairs}}}$,

$df = N_{\text{pairs}} - 2$ for all formulas

Coefficient of Determination

r^2

Spearman Rank-Order Correlation Coefficient

$r_s = 1 - \left[\dfrac{6\Sigma D^2}{(N_{\text{pairs}})(N^2_{\text{pairs}} - 1)}\right]$

Regression

Equation of a Straight Line

$Y = bX + a$

Slope of a Straight Line

$b = \dfrac{\text{Change in value of } Y}{\text{Change in value of } X} = \dfrac{Y_2 - Y_1}{X_2 - X_1}$

Equation of Least-Squares Linear Regression Line

$Y' = bX + a$

Slope of Least-Squares Linear Regression Line

Deviational formula: $b = \dfrac{\Sigma(X - \overline{X})(Y - \overline{Y})}{\Sigma(X - \overline{X})^2}$

Cross products formula: $b = \dfrac{CP_{XY}}{SS_X}$

Correlation formula: $b = r\left(\dfrac{s_Y}{s_X}\right)$

Y-Intercept of Least-Squares Linear Regression Line

$a = \overline{Y} - b\overline{X}$

Standard Error of Estimate

Definitional formula: $s_{Y\cdot X} = \sqrt{\dfrac{SS_{\text{Residual}}}{N_{\text{pairs}} - 2}}$ or $s_{Y\cdot X} = \sqrt{\dfrac{\Sigma(Y - Y')^2}{N_{\text{pairs}} - 2}}$

Correlation formula: $s_{Y\cdot X} = s_Y\sqrt{\left[\dfrac{N_{\text{pairs}} - 1}{N_{\text{pairs}} - 2}\right](1 - r_{\text{obs}}^2)}$

Nonparametric Tests

Chi-Square Test for Independence

$$\chi^2 = \Sigma\Sigma\frac{(O_{rc} - E_{rc})^2}{E_{rc}}, \quad df = (r - 1)(c - 1), \text{ or simplified formula,}$$

$$\chi^2 = \sum\frac{(O - E)^2}{E}, df = (r - 1)(c - 1)$$

$$\text{Expected frequency of a cell} = \frac{\text{(Row marginal for cell)(Column marginal for cell)}}{\text{Total number of responses}}$$

Chi-Square Test for Goodness of Fit

$$\chi^2 = \sum\frac{(O - E)^2}{E}, df = c - 1$$

Mann-Whitney *U* Test

$$U_{A_1} = n_{A_1}n_{A_2} + \frac{n_{A_1}(n_{A_1} + 1)}{2} - \Sigma R_{A_1}$$

$$U_{A_2} = n_{A_1}n_{A_2} + \frac{n_{A_2}(n_{A_2} + 1)}{2} - \Sigma R_{A_2}$$

or

$$U_{A_2} = n_{A_1}n_{A_2} - U_{A_1}$$

Wilcoxon Signed-Ranks Test

T = smaller of $\Sigma(-R)$ or $\Sigma(+R)$